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Eikonal cross sections from a time-dependent view†

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Abstract. Differential cross sections are written in terms of a time autocorrelation function involving the transition operator. Such a formulation is useful when time-dependent approximations are considered. Eikonal theory provides a good example. Using the time autocorrelation function representation, a general eikonal differential cross section is obtained which is parametrised by a momentum. If this parameter is taken as the incoming momentum, then the resulting eikonal differential cross section is found to be complementary to the standard Glauber total cross section, that is, the angular average of the differential cross section is identical to the total cross section. Weak potential limits of these cross sections are Mott cross sections. On the other hand, if the parameter is taken as the average of the incoming and outgoing momenta, then an eikonal differential cross section is produced whose complementary total cross section is not of Glauber form. However, the weak potential limits of these cross sections are the Born cross sections.

1. Introduction

In the description of binary quantal scattering events, an often used approximation is eikonal theory (see, for example, Glauber 1959, Schiff 1968, Joachain and Quigg 1974, Newton 1966, Bransden 1970). The eikonal wavefunction is usually obtained either as a path integral approximation to the phase (Schiff 1968), or as a high-energy approximation to the free Green function (Joachain and Quigg 1974). Differential and total cross sections are then obtained by reducing the eikonal wavefunction to the Glauber wavefunction (Glauber 1959). However, the Glauber differential and Glauber total cross sections are *not* complementary, that is, the angular average of the Glauber differential cross section is not identical to the Glauber total cross section. Complementarity is only obtained when further approximations are made to this angular average (see, for example, Glauber 1959).

The purpose of this paper is to remedy this situation, that is, to obtain an eikonal differential cross section that is the complement of the Glauber total cross section without recourse to further approximations. To do so, a class of eikonal differential cross sections is obtained. It is then interesting to consider the weak potential limits of these cross sections, since the weak potential limit of the eikonal wavefunction is the Born wavefunction. It is found that the Glauber total cross section and its eikonal differential complement reduce to Mott (1931) cross sections and *not* to Born cross sections.

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To obtain this class of eikonal differential cross sections, it is convenient to use a time autocorrelation expression for the exact differential cross section as a starting point, rather than the standard static form (square of a matrix element of the transition operator). Such an expression is obtained in § 2. The derivation of this time autocorrelation form is based upon the generalised cross section description of the scattering event given by Snider (1975) and Coombe *et al* (1975).

Having a time autocorrelation function involving the transition operator then requires an eikonal transition operator to be defined for the expression to be of use. In § 3 an eikonal transition operator is defined using the eikonal Møller operator obtained by Turner (1980). This eikonal Møller operator is parametrised by a momentum which may be varied. Thus the class of eikonal differential cross sections which are obtained from using this eikonal transition operator in the autocorrelation function is determined by the momentum parameter. In § 4 this momentum is taken to be the initial momentum. The eikonal differential cross section thus defined is complementary to the standard Glauber total cross section. In the weak potential limit it reduces to the complement of the Mott (1931) total cross section, thus defining a differential Mott cross section. Finally, in § 5, the momentum parameter is taken as the average of the incoming and outgoing momenta. The total cross section complementary to this eikonal differential cross section does not have the simple Glauber form. However, their weak potential limits are the complementary Born cross sections.

2. Differential cross section as a time correlation function

To obtain a time correlation expression for the differential cross section, it is convenient to start with the generalised cross section (Snider 1975, Coombe *et al* 1975)

$$\sigma_{\text{gen}}(\boldsymbol{p}''\to\hat{\boldsymbol{R}}) = \mathrm{Tr}(|\hat{\boldsymbol{R}}\rangle\langle\hat{\boldsymbol{R}}|)(-\mathrm{i}\mathscr{T})(|\boldsymbol{p}''\rangle\mu h^3 p''^{-1}\langle\boldsymbol{p}''|). \tag{2.1}$$

Here, $|\boldsymbol{p}''\rangle(\mu h^3/p'')\langle\boldsymbol{p}''|$ is a plane wave density which is transformed into a spherical density $|\hat{\boldsymbol{R}}\rangle\langle\hat{\boldsymbol{R}}|$ by the transition superoperator

$$\mathscr{T} = \mathscr{V}\Omega_L. \tag{2.2}$$

The reduced mass is μ, the incoming momentum is $\boldsymbol{p}''$ and the observation direction is $\hat{\boldsymbol{R}}$. Involved in the expression for the transition superoperator are the potential superoperator $\mathscr{V}$, $\hbar^{-1}$ times the commutator with the potential energy V, and the Møller superoperator,

$$\Omega_L = \lim_{t\to -} \exp(\mathrm{i}\mathscr{L}t)\exp(-\mathrm{i}\mathscr{K}t). \tag{2.3}$$

This Møller superoperator contains the full Liouville or von Neumann superoperator $\mathscr{L}$ ($\hbar^{-1}$ times the commutator with the Hamiltonian $H = K + V$) and the drift or kinetic superoperator $\mathscr{K}$ ($\hbar^{-1}$ times the commutator with the kinetic energy K).

The expression for the generalised cross section can be related to the transition operator forms of the differential and total cross sections. Indeed, it is just such a relation that produces the time correlation expression for the differential cross section. To obtain the transition operator forms, use is made of the relation (Jauch *et al* 1968, Turner 1977)

$$\Omega_L\rho = \Omega^{(+)}\rho\,\Omega^{(+)\dagger} \tag{2.4}$$

between the Møller superoperator and the Møller operator

$$\Omega^{(+)} = \lim_{t\to-\infty} \exp(iHt/\hbar)\exp(-iKt/\hbar). \tag{2.5}$$

Here ρ is an operator. In particular, using equation (2.4) in equation (2.1) leads to the expression

$$\sigma_{\text{gen}}(\boldsymbol{p}''\to\hat{\boldsymbol{R}}) = -(4\pi h^2\mu/p'')\,\text{Im}\int_0^\infty \mathrm{d}p\,p^2\langle p\hat{\boldsymbol{R}}|\Omega^{(+)}|\boldsymbol{p}''\rangle\langle\boldsymbol{p}''|t_{\text{op}}^\dagger|p\hat{\boldsymbol{R}}\rangle, \tag{2.6}$$

for the generalised cross section that contains the Møller operator and the adjoint of the transition operator,

$$t_{\text{op}} = V\Omega^{(+)}. \tag{2.7}$$

Differential and total cross sections can be obtained from equation (2.6) when the Lippmann–Schwinger equation,

$$\Omega^{(+)} = 1_{\text{op}} - (i/\hbar)\int_{-\infty}^{0} \mathrm{d}s\,t_{\text{op}}(s), \tag{2.8}$$

is used. Here the time-dependent transition operator is

$$t_{\text{op}}(s) = \exp(i\mathcal{K}s)t_{\text{op}} = \exp(iKs/\hbar)t_{\text{op}}\exp(-iKs/\hbar). \tag{2.9}$$

Equation (2.8) is the time integral form (see, for example, Turner 1980) of the usual energy-dependent Lippmann–Schwinger equation relating the Møller operator and the transition operator. Using equation (2.8) in equation (2.6) gives the relation (Snider 1975, Coombe *et al* 1975)

$$\sigma_{\text{gen}}(\boldsymbol{p}''\to\hat{\boldsymbol{R}}) = \sigma(\boldsymbol{p}''\to\hat{\boldsymbol{R}}) - \sigma_{\text{tot}}(\boldsymbol{p}'')\delta^{(2)}(\hat{\boldsymbol{R}} - \boldsymbol{p}'') \tag{2.10}$$

between the generalised cross section, the differential cross section,

$$\sigma(\boldsymbol{p}''\to\hat{\boldsymbol{R}}) = (2\pi h\mu)^2|\langle p''\hat{\boldsymbol{R}}|t_{\text{op}}|\boldsymbol{p}''\rangle|^2, \tag{2.11}$$

and the total cross section

$$\sigma_{\text{tot}}(\boldsymbol{p}'') = -(4\pi h^2\mu/p'')\,\text{Im}\langle\boldsymbol{p}''|t_{\text{op}}|\boldsymbol{p}''\rangle = \int \mathrm{d}\hat{\boldsymbol{R}}\,\sigma(\boldsymbol{p}''\to\hat{\boldsymbol{R}}). \tag{2.12}$$

Equations (2.11) and (2.12) are the standard transition operator expressions for the differential and total cross sections (see, for example, Newton 1966).

Equation (2.11) was obtained by explicitly evaluating the time integral which resulted upon use of equation (2.8) in equation (2.6). However, if this time integral is not explicitly performed, then the time correlation expression

$$\sigma(\boldsymbol{p}''\to\hat{\boldsymbol{R}}) = (8\pi^2 h\mu/p'')\int_0^\infty \mathrm{d}p\,p^2\,\text{Im}\,i\int_{-\infty}^{0}\mathrm{d}s\langle p\hat{\boldsymbol{R}}|t_{\text{op}}(s)|\boldsymbol{p}''\rangle\langle\boldsymbol{p}''|t_{\text{op}}^\dagger|p\hat{\boldsymbol{R}}\rangle \tag{2.13}$$

for the differential cross section is obtained. It involves an autocorrelation function in the transition operator with time dynamics generated by the kinetic or drift superoperator $\mathcal{K}$, see equation (2.9). The advantage of this expression for differential cross sections over the standard formula, equation (2.11), occurs when time-dependent approximations to the transition operator are considered. A particularly good example of the advantage of the time correlation expression is eikonal theory.

3. General eikonal cross section

To obtain a general eikonal differential cross section from the time correlation expression, it is necessary to evaluate eikonal approximations to the transition operator at time zero and time s. This requires an eikonal Møller operator. Turner (1980) has recently presented a derivation of the eikonal Møller operator based upon the interaction picture representation. In particular, the Møller operator, equation (2.5), becomes

$$\Omega^{(+)} = T\exp\left((-\mathrm{i}/\hbar)\int_{-\infty}^{0} \mathrm{d}s\, V_K(s)\right) \equiv \Omega^{(+)}(0) \tag{3.1}$$

in the interaction picture. The zero in $\Omega^{(+)}(0)$ refers to the upper limit of integration, while T is the Dyson (1949) chronological operator. This representation of the Møller operator involves the interaction picture potential,

$$V_K(s) = \exp(\mathrm{i}\mathcal{K}s)\,V, \tag{3.2}$$

which can be *exactly* written as

$$V_K(s) = h^{-3}\int \mathrm{d}\boldsymbol{z}\,\mathrm{d}\boldsymbol{x}\,\mathrm{d}\boldsymbol{p}\,\exp[-\mathrm{i}\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{z})/\hbar]|\boldsymbol{z}\rangle V[\tfrac{1}{2}(\boldsymbol{x}+\boldsymbol{z})+\boldsymbol{p}s/\mu]\langle\boldsymbol{x}|. \tag{3.3}$$

The eikonal potential,

$$V_K(s|\boldsymbol{P})^{\mathrm{EA}} = V(\boldsymbol{r}_{\mathrm{op}}+\boldsymbol{P}s/\mu), \tag{3.4}$$

and the eikonal Møller operator,

$$\Omega^{(+)}(0|\boldsymbol{P})^{\mathrm{EA}} = \exp\left((-\mathrm{i}/\hbar)\int_{-\infty}^{0}\mathrm{d}s\, V(\boldsymbol{r}_{\mathrm{op}}+\boldsymbol{P}s/\mu)\right), \tag{3.5}$$

are obtained when the trajectory $\frac{1}{2}(\boldsymbol{x}+\boldsymbol{z})+\boldsymbol{p}s/\mu$ is replaced by the trajectory $\frac{1}{2}(\boldsymbol{x}+\boldsymbol{z})+\boldsymbol{P}s/\mu$. This latter trajectory contains a given but non-unique momentum $\boldsymbol{P}$. In the next two sections two particular choices for this momentum are discussed. However, for the present, this momentum is not specified.

Making use of equations (2.7) and (3.5), the eikonal transition operator at time zero becomes

$$t_{\mathrm{op}}(0|\boldsymbol{P})^{\mathrm{EA}} = V\Omega^{(+)}(0|\boldsymbol{P})^{\mathrm{EA}}. \tag{3.6}$$

Since this operator is diagonal in position space, its exact time-dependent motion is

$$\begin{aligned}\exp(\mathrm{i}\mathcal{K}s)&t_{\mathrm{op}}(0|\boldsymbol{P})^{\mathrm{EA}}\\ &= h^{-3}\int \mathrm{d}\boldsymbol{x}\,\mathrm{d}\boldsymbol{z}\,\mathrm{d}\boldsymbol{p}\,\exp[-\mathrm{i}\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{z})/\hbar]|\boldsymbol{z}\rangle V[\tfrac{1}{2}(\boldsymbol{x}+\boldsymbol{z})+\boldsymbol{p}s/\mu]\\ &\quad\times\exp\left((-\mathrm{i}/\hbar)\int_{-\infty}^{0}\mathrm{d}s'\,V[\tfrac{1}{2}(\boldsymbol{x}+\boldsymbol{z})+\boldsymbol{p}s/\mu+\boldsymbol{P}s'/\mu]\right)\langle\boldsymbol{x}|.\end{aligned} \tag{3.7}$$

As with the Møller operator, this time-dependent transition operator can be diagonalised in the position representation by again replacing the trajectory $\frac{1}{2}(\boldsymbol{x}+\boldsymbol{z})+\boldsymbol{p}s/\mu$ with the trajectory $\frac{1}{2}(\boldsymbol{x}+\boldsymbol{z})+\boldsymbol{P}s/\mu$. Making this replacement defines the time-dependent eikonal transition operator,

$$t_{\rm op}(s|\boldsymbol{P})^{\rm EA} = V(\boldsymbol{r}_{\rm op} + \boldsymbol{P}s/\mu)\Omega^{(+)}(s|\boldsymbol{P})^{\rm EA}, \tag{3.8}$$

which is consistent with the time-dependent eikonal Møller operator.

The general eikonal differential cross section,

$$\sigma(\boldsymbol{p}'' \to \hat{\boldsymbol{R}}|\boldsymbol{P})^{\rm EA} = (8\pi^2 h\mu/p'') \int_0^\infty {\rm d}p\, p^2 {\rm Im\, i} \int_{-\infty}^0 {\rm d}s \langle p\hat{\boldsymbol{R}}|t_{\rm op}(s|\boldsymbol{P})^{\rm EA}|\boldsymbol{p}''\rangle \times \langle \boldsymbol{p}''\, t_{\rm op}(0\, \boldsymbol{P})^{\rm EA\dagger}\, p\hat{\boldsymbol{R}}\rangle, \tag{3.9}$$

is obtained when the eikonal transition operator, equation (3.8), is used in the time correlation expression for the differential cross section, equation (2.13). This equation is termed the general eikonal differential cross section since it contains an as yet unspecified momentum $\boldsymbol{P}$. Different results will arise upon different choices for this momentum. Two such choices will now be considered. One choice is the forward direction $\boldsymbol{p}''$ which is the standard choice for eikonal approximations (see, for example, Schiff 1968) while the other choice is the average momentum $\frac{1}{2}(p\hat{\boldsymbol{R}} + \boldsymbol{p}'')$ (see, for example, Newton 1966).

4. Initial momentum situation

Here the momentum $\boldsymbol{P}$ is taken as the initial momentum $\boldsymbol{p}''$ of the projectile. Thus the eikonal differential cross section, equation (3.9), becomes

$$\sigma(\boldsymbol{p}'' \to \hat{\boldsymbol{R}}|\boldsymbol{p}'')^{\rm EA} = (8\pi^2 h\mu/p'') \int_0^\infty {\rm d}p\, p^2 {\rm Im\, i} \int_{-\infty}^0 {\rm d}s \langle p\hat{\boldsymbol{R}}|t_{\rm op}(s|\boldsymbol{p}'')^{\rm EA}|\boldsymbol{p}''\rangle \times \langle \boldsymbol{p}''|t_{\rm op}(0|\boldsymbol{p}'')^{\rm EA\dagger}|p\hat{\boldsymbol{R}}\rangle. \tag{4.1}$$

This eikonal differential cross section is consistent with the standard definition of the Glauber (1959) total cross section, that is, the angular integral of this eikonal cross section is equal to the Glauber total cross section. This result is easily demonstrated by performing the angular integral, that is ($\boldsymbol{p} = p\hat{\boldsymbol{R}}$)

$$\begin{aligned}\int {\rm d}\hat{\boldsymbol{R}}\, \sigma(\boldsymbol{p}'' \to \hat{\boldsymbol{R}}|\boldsymbol{p}'')^{\rm EA} &= (8\pi^2 h\mu/p'')\, {\rm Im\, i} \int_{-\infty}^0 {\rm d}s \int {\rm d}\boldsymbol{p} \langle \boldsymbol{p}''|t_{\rm op}(0|\boldsymbol{p}'')^{\rm EA\dagger}|\boldsymbol{p}\rangle \\ &\quad \times \langle \boldsymbol{p}|t_{\rm op}(s|\boldsymbol{p}'')^{\rm EA}|\boldsymbol{p}''\rangle \\ &= (8\pi^2 h\mu/p''){\rm Im\, i} \int_{-\infty}^0 {\rm d}s \langle \boldsymbol{p}''|t_{\rm op}(0|\boldsymbol{p}'')^{\rm EA\dagger} t_{\rm op}(s|\boldsymbol{p}'')^{\rm EA}|\boldsymbol{p}''\rangle \\ &= (8\pi^2 \mu/h^2 p''){\rm Im\, i} \int_{-\infty}^0 {\rm d}s \int {\rm d}\boldsymbol{r} V(\boldsymbol{r}) V(\boldsymbol{r} + \boldsymbol{p}''s/\mu) \\ &\quad \times \exp\left((-{\rm i}/\hbar) \int_0^s {\rm d}s' V(\boldsymbol{r} + \boldsymbol{p}''s'/\mu)\right).\end{aligned} \tag{4.2}$$

Now, taking the imaginary part and performing the time integral gives

$$\int {\rm d}\hat{\boldsymbol{R}}\, \sigma(\boldsymbol{p}'' \to \hat{\boldsymbol{R}}|\boldsymbol{p}'')^{\rm EA} = (4\pi\mu/hp'') \int {\rm d}\boldsymbol{r}\ V(\boldsymbol{r}) \sin\left(\hbar^{-1} \int_{-\infty}^0 {\rm d}s' V(\boldsymbol{r} + \boldsymbol{p}''s'/\mu)\right). \tag{4.3}$$

Finally, writing the position $\boldsymbol{r}$ in cylindrical coordinates, i.e. $\boldsymbol{r}=\boldsymbol{b}+z\hat{p}''$, $\boldsymbol{b}\cdot\hat{p}''=0$, this total cross section becomes

$$\int \mathrm{d}\hat{R}\, \sigma(\boldsymbol{p}''\rightarrow\hat{R}|\boldsymbol{p}'')^{\mathrm{EA}}=(4\pi\mu/hp'')\int_{-\infty}^{\infty}\mathrm{d}z\int \mathrm{d}^{(2)}\boldsymbol{b}V(\boldsymbol{b}+z\hat{p}'')$$

$$\times\sin\left((\mu/\hbar p'')\int_{-\infty}^{z}\mathrm{d}z'V(\boldsymbol{b}+z'\hat{p}'')\right)$$

$$=-2\int \mathrm{d}^{(2)}\boldsymbol{b}[\cos\chi(\boldsymbol{b})-1]\equiv\sigma_{\mathrm{tot}}^{\mathrm{GL}}, \tag{4.4}$$

where $\chi(\boldsymbol{b})$ is the standard Glauber (1959) phase,

$$\chi(\boldsymbol{b})=(\mu/\hbar p'')\int_{-\infty}^{\infty}\mathrm{d}z\; V(\boldsymbol{b}+z\hat{p}''). \tag{4.5}$$

Equation (4.4) is the standard Glauber total cross section. Thus, the eikonal differential cross section defined by equation (4.1) is the differential cross section that should be associated with the total Glauber cross section. It is not equal to the Glauber (1959) differential cross section. In fact, the Glauber differential cross section is an approximation to equation (4.1). In particular, if the time integral of the eikonal transition operator is taken to be its initial ($s=0$) value, if the final momentum p is taken as p'' and if the momentum transfer, $\boldsymbol{p}''-p''\hat{\boldsymbol{R}}$, is assumed to have no component in the incoming direction, then the Glauber (1959) differential cross section

$$\sigma(\boldsymbol{p}''\rightarrow\hat{R})^{\mathrm{GL}}=(p''/\hbar)^2\left|\int \mathrm{d}^{(2)}\boldsymbol{b}[\exp(-\mathrm{i}\chi(\boldsymbol{b}))-1]\exp[\mathrm{i}(\boldsymbol{p}''-p''\hat{R})\cdot\boldsymbol{b}/\hbar]\right|^2 \tag{4.6}$$

is obtained. On the other hand, this Glauber cross section can be obtained from the standard transition operator expression for the differential cross section, equation (2.11), by replacing the transition operator with the initial eikonal transition operator, equation (3.6), and by assuming that the momentum transfer has no component in the initial direction. To obtain the total Glauber cross section from the Glauber differential cross section requires a further approximation; see, for example, Glauber (1959).

It is of interest to consider the weak potential limit of this eikonal differential cross section and its complement, the total Glauber cross section. In particular, the Glauber total cross section reduces to the Mott (1931) total cross section, that is,

$$\sigma_{\mathrm{tot}}^{\mathrm{GL}}=-2\int \mathrm{d}^{(2)}\boldsymbol{b}[\cos\chi(\boldsymbol{b})-1]\sim\int \mathrm{d}^{(2)}\boldsymbol{b}\,\chi(\boldsymbol{b})^2$$

$$\sim\int \mathrm{d}^{(2)}\boldsymbol{b}\left|(\mu/\hbar p'')\int_{-\infty}^{\infty}\mathrm{d}z\; V(\boldsymbol{b}+z\hat{p}'')\right|^2\equiv\sigma_{\mathrm{tot}}^{\mathrm{Mott}}, \tag{4.7}$$

where the last form is recognised as the Mott result. Since the Glauber total cross section reduces to the Mott total cross section, the eikonal differential cross section should reduce to a differential cross section which is the complement of Mott's result. Indeed this is what happens for the weak potential limit of equation (4.1), namely,

$$\sigma(\boldsymbol{p}''\rightarrow\hat{R}|\boldsymbol{p}'')^{\mathrm{EA}}\sim(8\pi^2h\mu/p'')\int_0^{\infty}\mathrm{d}p\,p^2\int_{-\infty}^{0}\mathrm{d}s\langle p\hat{R}|V(\boldsymbol{r}_{\mathrm{op}}+\boldsymbol{p}''s/\mu)|p''\rangle\langle \boldsymbol{p}''|V|p\hat{R}\rangle$$

$$\equiv\sigma(\boldsymbol{p}''\rightarrow\hat{R})^{\mathrm{Mott}} \tag{4.8}$$

(where the time integral of the matrix elements is real) is the complement of the total Mott cross section. It is thus termed the Mott differential cross section. As with the eikonal differential cross section, straightforward angular integration of equation (4.8) reproduces equation (4.7), thus demonstrating the complementary nature of these two cross sections.

By choosing the momentum $\boldsymbol{P}$ to be the incoming momentum $\boldsymbol{p}''$, the resulting eikonal differential cross section, equation (4.1), is found to be the complement of the usual Glauber total cross section. When the potential is assumed to be weak, the complementary Mott cross sections were obtained rather than the complementary Born approximations. The question then arises as to what choice for $\boldsymbol{P}$ will lead to the Born approximations in the weak potential limit. As demonstrated in the next section, the choice is the average momentum $\frac{1}{2}(p\hat{\boldsymbol{R}}+\boldsymbol{p}'')$.

5. Average momentum situation

When $\boldsymbol{P}$ is chosen as the average of the incoming and outgoing momenta, the eikonal differential cross section becomes

$$\begin{aligned}\sigma[\boldsymbol{p}''\to\hat{\boldsymbol{R}}|\tfrac{1}{2}(p\hat{\boldsymbol{R}}+\boldsymbol{p}'')]^{\mathrm{EA}} &= (8\pi^2 h\mu/p'')\int_0^\infty \mathrm{d}p\, p^2 \mathrm{Im}\, \mathrm{i}\int_{-\infty}^0 \mathrm{d}s \langle p\hat{\boldsymbol{R}}|t_{\mathrm{op}}[s|\tfrac{1}{2}(p\hat{\boldsymbol{R}}+\boldsymbol{p}'')]^{\mathrm{EA}}|\boldsymbol{p}''\rangle \\ &\quad\times\langle \boldsymbol{p}''\ t_{\mathrm{op}}[0\ \tfrac{1}{2}(p\hat{\boldsymbol{R}}+\boldsymbol{p}'')]^{\mathrm{EA}\dagger}\ p\hat{\boldsymbol{R}}\rangle,\end{aligned} \tag{5.1}$$

a form which differs from equation (4.1) only in the choice of $\boldsymbol{P}$. However, this is a crucial difference. For example, the total cross section,

$$\sigma_{\mathrm{tot}} = \int \mathrm{d}\hat{\boldsymbol{R}}\, \sigma[\boldsymbol{p}''\to\hat{\boldsymbol{R}}|\tfrac{1}{2}(p\hat{\boldsymbol{R}}+\boldsymbol{p}'')]^{\mathrm{EA}}, \tag{5.2}$$

complementary to equation (5.1) does not take on the simple Glauber form, as the resulting $\boldsymbol{p}(=p\hat{\boldsymbol{R}})$ integration is far from trivial. Furthermore, the weak potential limits of these cross sections are the complementary Born cross sections and not the Mott cross sections. In particular, equation (5.1) becomes

$$\begin{aligned}\sigma[\boldsymbol{p}''\to\hat{\boldsymbol{R}}|\tfrac{1}{2}(p\hat{\boldsymbol{R}}+\boldsymbol{p}'')]^{\mathrm{EA}} &\sim (8\pi^2 h\mu/p'')\int_0^\infty \mathrm{d}p\, p^2\int_{-\infty}^0 \mathrm{d}s \\ &\quad\times\langle p\hat{\boldsymbol{R}}\ V[\boldsymbol{r}_{\mathrm{op}}+\tfrac{1}{2}(p\hat{\boldsymbol{R}}+\boldsymbol{p}'')s/\mu]\ \boldsymbol{p}''\rangle\langle \boldsymbol{p}''\ V\ p\hat{\boldsymbol{R}}\rangle \\ &\equiv \sigma(\boldsymbol{p}''\to\hat{\boldsymbol{R}})^{\mathrm{Born}}\end{aligned} \tag{5.3}$$

in the weak potential limit. To obtain equation (5.3), the time integral of the matrix elements has been recognised as being real. Equation (5.3) is the time correlation form of the Born differential cross section (see, for example, Turner and Snider 1976). It is interesting to note that the only difference between the Mott and Born differential cross sections is the direction of the straight line trajectory. In the Mott case the trajectory is such that the particle goes straight through, i.e. no deflection, while in the Born case it follows a straight path directed along the average of the incoming and outgoing momenta, i.e. some deflection.

6. Discussion

Differential cross sections have been expressed in terms of a time autocorrelation function of the transition operator. The advantage of such a formulation over the usual static representation lies in its applicability to time-dependent approximations. This has been exemplified by the use of eikonal theory. In the time correlation function representation a general eikonal differential cross section, equation (3.9), is obtained which is parametrised by an adjustable momentum $\boldsymbol{P}$.

When this momentum is taken to be the initial momentum $\boldsymbol{p}''$, then the said eikonal differential cross section is found to be the complement of the standard Glauber total cross section. By complement it is meant that the angular average of the differential cross section is identical to the total cross section. The weak potential limits of these cross sections are equal to the complementary Mott cross sections. On the other hand, if the momentum $\boldsymbol{P}$ is taken to be the average of the incoming and outgoing momenta, then an eikonal differential approximation is obtained whose complementary total cross section does not have the usual Glauber form. However, the cross sections defined with this momentum reduce to the complementary Born approximations in the weak potential limit.

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